GCSE

185/09

MATHEMATICS
HIGHER TIER
PAPER 1

A.M. TUESDAY, 9 November 2010

2 hours

CALCULATORS ARE NOT TO BE USED FOR THIS PAPER

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take π as 3·14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.
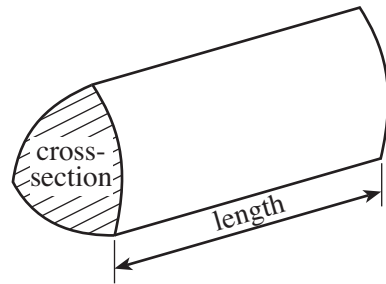
The number of marks is given in brackets at the end of each question or part-question.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1	5	
2	9	
3	9	
4	5	
5	7	
6	4	
7	8	
8	10	
9	6	
10	6	
11	2	
12	5	
13	9	
14	11	
15	4	
TOTAL MARK		

2

Formula List

Volume of prism = area of cross-section × length

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$
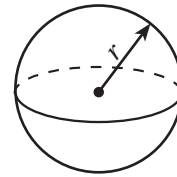
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl
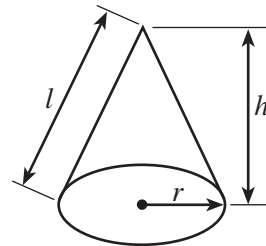
In any triangle *ABC*

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\dfrac{1}{2}ab\sin C$
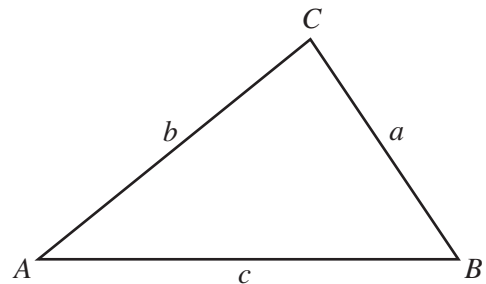
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Standard Deviation

Standard deviation for a set of numbers

$x_1, x_2, \ldots, x_n$, having a mean of $\bar{x}$ is given by

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n}} \quad \text{or } s = \sqrt{\frac{\sum x^2}{n} - \left\{\frac{\sum x}{n}\right\}^2}$$

(185-09)

1. The number of visitors to a historical site and the total donation given were recorded each weekend for 6 weeks. The table below shows the results.

Number of visitors	90	140	10	60	100	180
Total donation, £	360	650	40	150	410	700

(a) On the graph paper provided draw a scatter diagram of these results.

[2]

Total donation, £
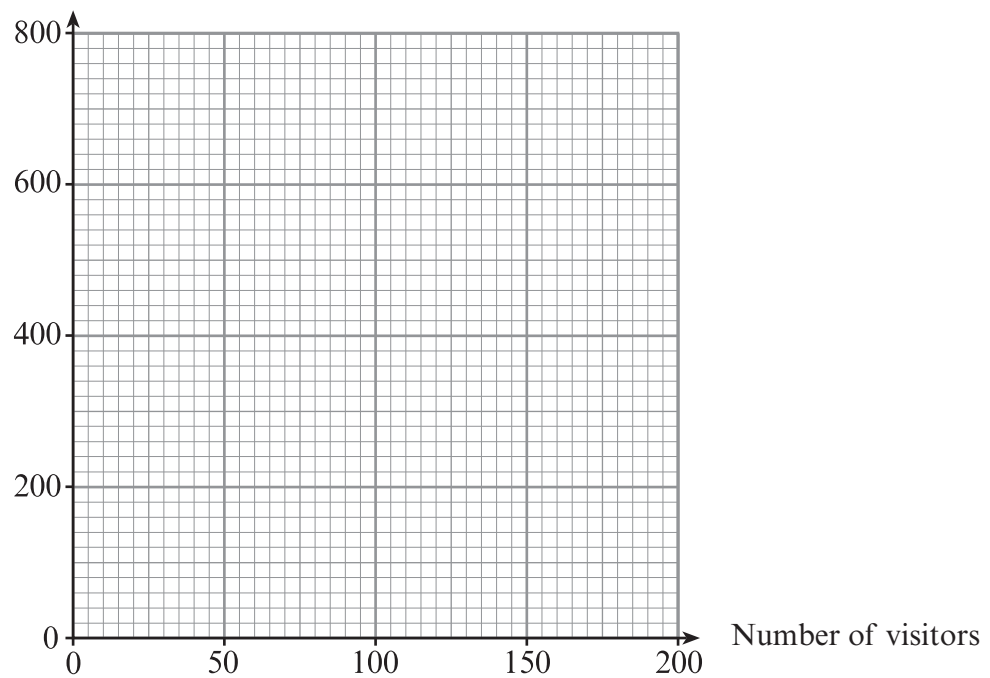
(b) Describe the correlation between the number of visitors and the total donation.

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[1]

(c) Draw, by eye, a line of best fit on your scatter diagram.

[1]

(d) Use your line of best fit to find an estimate for the number of visitors to the historical site on a weekend when the total donation was £500.

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[1]

Turn over.

2. *(a)* On the grid below, draw the enlargement of the given shape using a scale factor of 2 and centre *O*.

[3]
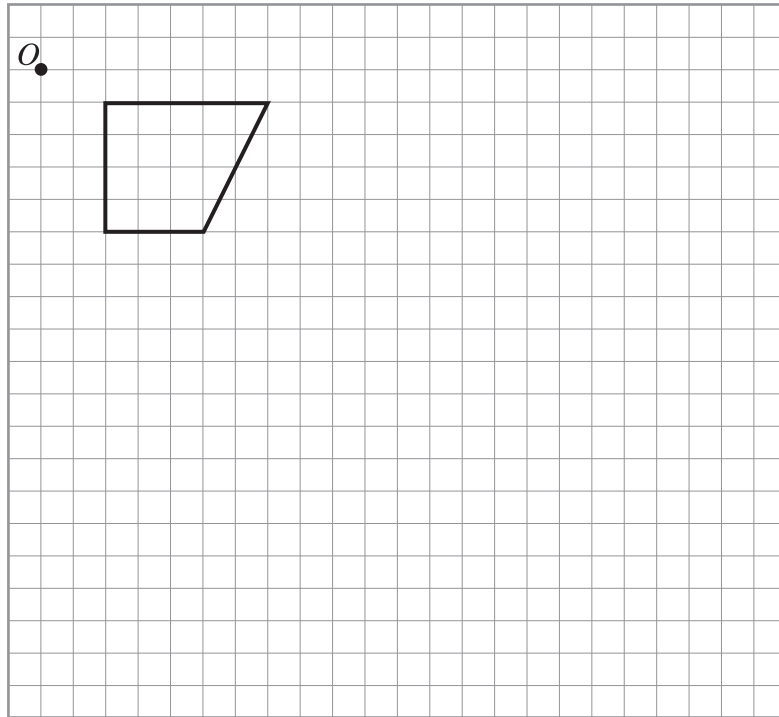
(b) Describe fully the transformation that transforms triangle *A* into triangle *B*.

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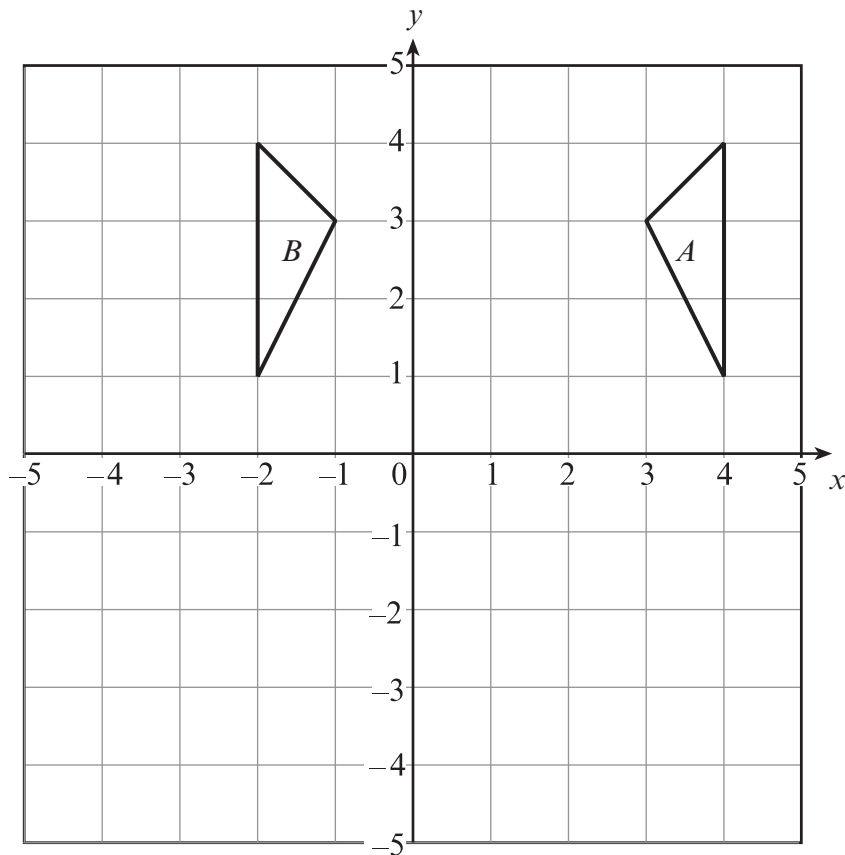
[2]

(c) Rotate the triangle *A* through 90° anticlockwise about the origin.

[2]
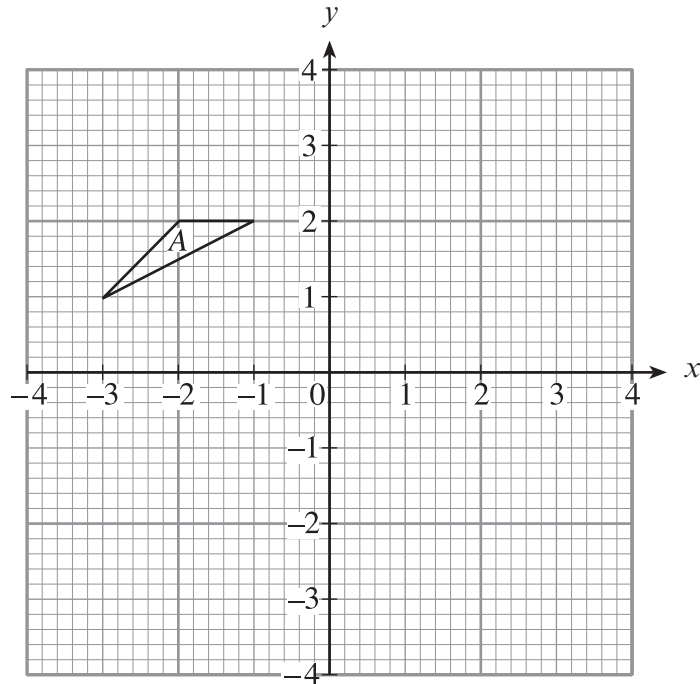
(d) Enlarge the square *PQRS* using centre (0, 0) by a scale factor of ½.

[2]

Turn over.
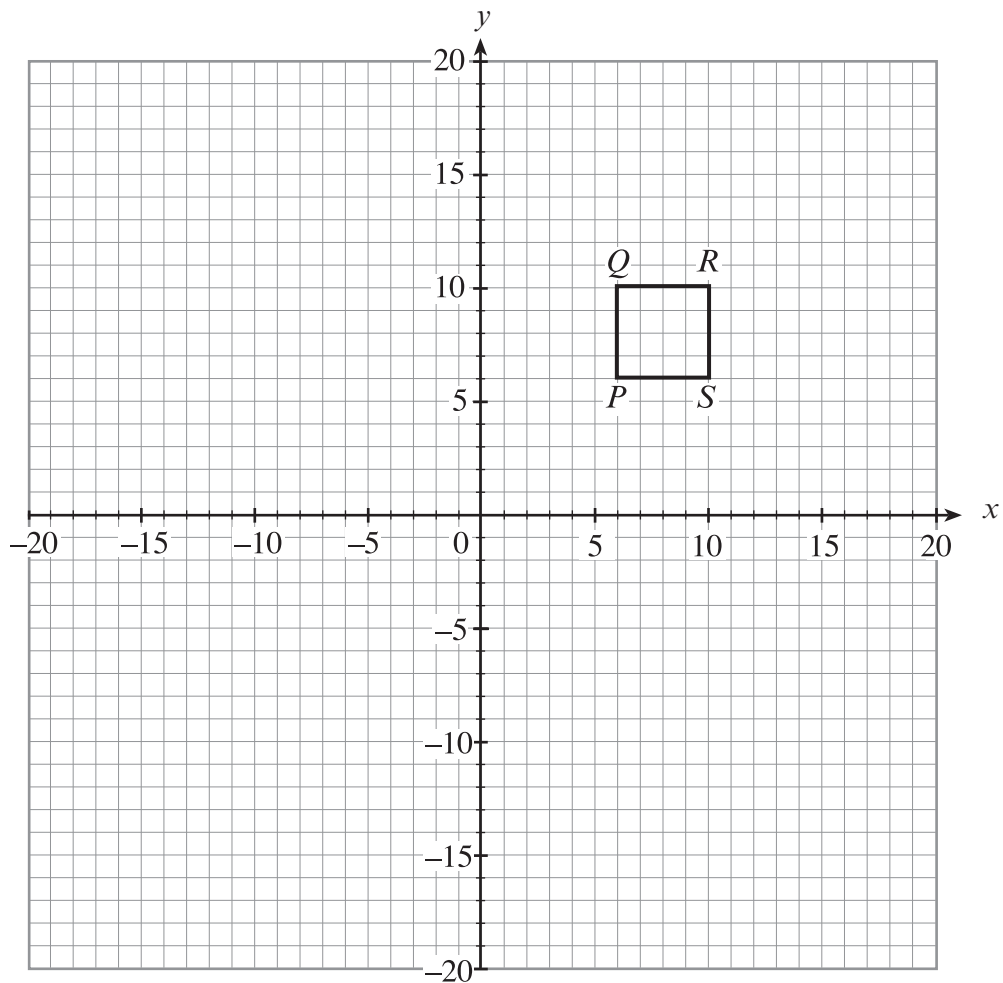
3. *(a)*
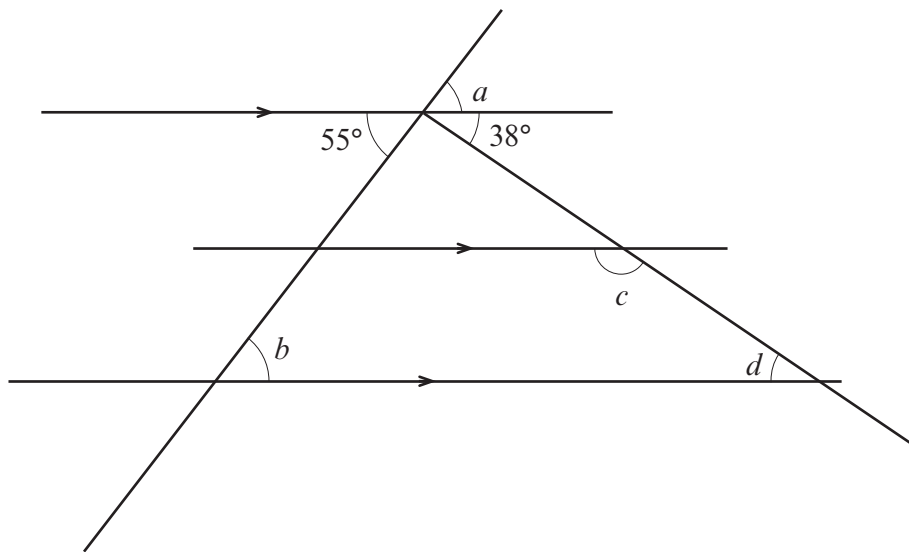
Diagram not drawn to scale

Find the size of each of the angles marked *a, b, c* and *d*.

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a = ... °   *b* = ... °   *c* = ... °   *d* = ... °

[4]

(b)   This plan is drawn to scale.

Scale: 1 cm represents 20 m

N

N

Y

X

0185
090007
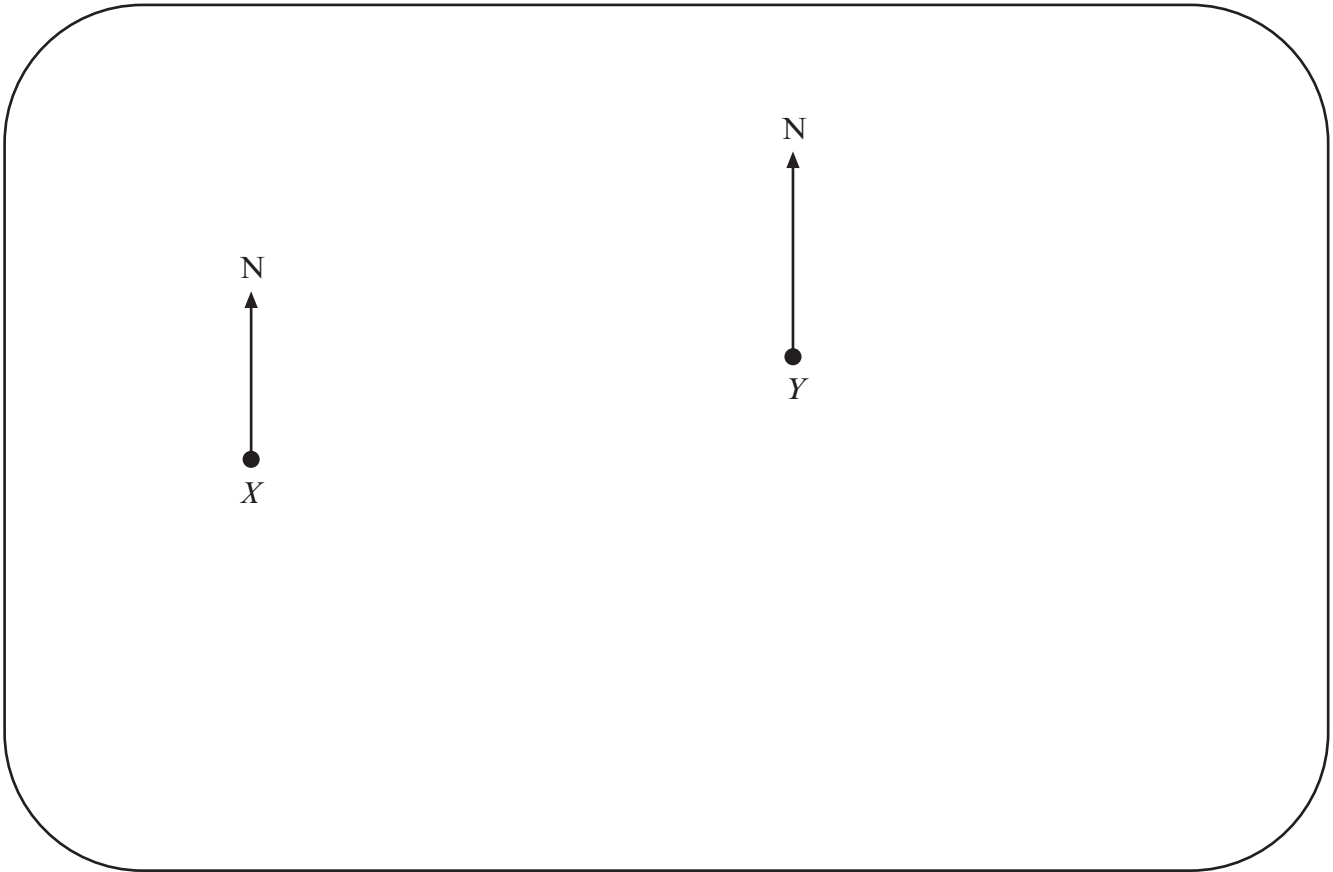
(i)   Write down the bearing of the point *X* from the point *Y*.

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(ii)   A point *Z* is to be plotted on the above plan.
The bearing of *Z* from *X* is 100°, and the bearing of *Z* from *Y* is 225°.
Find and mark the position of *Z* on the above plan.

(iii)   How far is *Z* from *X* in metres?

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[5]

Turn over.

4.
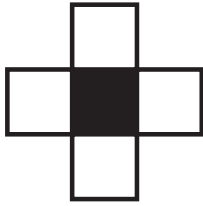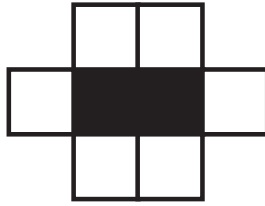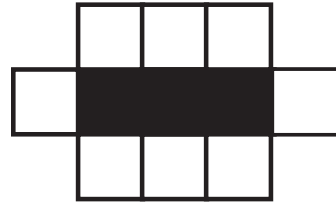
| Pattern 1 | Pattern 2 | Pattern 3 |

Complete the following table.

Pattern number	Number of black squares	Number of white squares
1	1	4
2	2	6
3	3	8

15		

100		

n		

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[5]

5. *(a)* Given that

$$234 \times 789 = 184626$$

find

(i) $23 \cdot 4 \times 7 \cdot 89$,

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(ii) $\dfrac{18462 \cdot 6}{2 \cdot 34}$,

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(iii) 117×789.

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[3]

(b) Share £385 in the ratio 2 : 9.

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[2]

(c) Estimate the value of $\dfrac{607 \cdot 45}{0 \cdot 2498}$.

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[2]

Turn over.

6. *(a)*
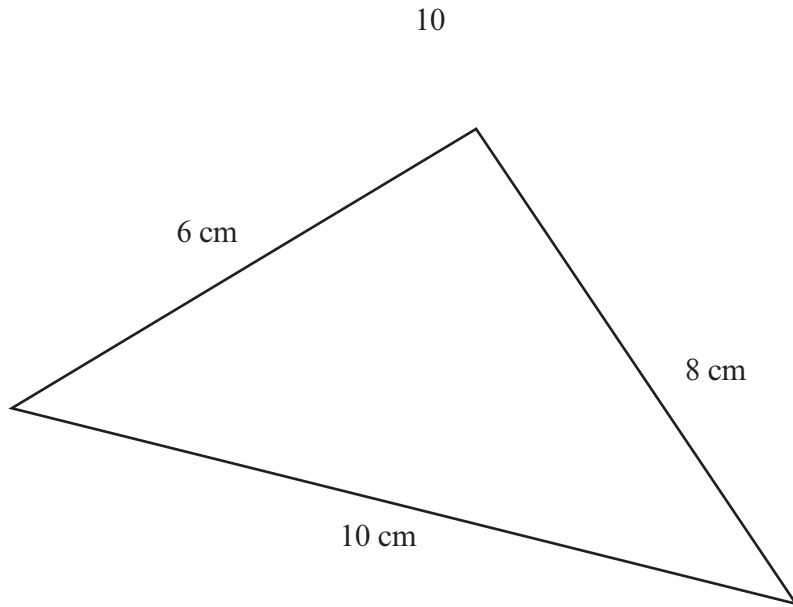
Diagram not drawn to scale

Show, by calculation, that the triangle drawn above is a right-angled triangle.

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[2]

(b) Calculate the area of a semicircle with a radius of 20 cm using $\pi = 3 \cdot 14$.

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[2]

7. *(a)* Three bananas and one apple cost a total of 65p.
Seven bananas and two apples cost a total of £1.49.
How much does one apple cost?

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[4]

(b) Solve $\dfrac{2x+3}{6} + \dfrac{x-1}{3} = \dfrac{7}{12}$.

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[4]

8. *(a)* Simplify $\dfrac{a^6 \times a^8}{a^7}$.

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[1]

(b) Expand $x(x + 2)$.

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[1]

(c) Make w the subject of the following formula.

$$6(w + y) = 10y - 7$$

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[3]

(d) Factorise $x^2 + x - 12$ and hence solve the equation $x^2 + x - 12 = 0$.

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[3]

(e) Solve $3n + 5 > 9 + n$.

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[2]

9. *(a)* Express 756 as a product of prime numbers in index form.

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[3]

(b) Evaluate the following. Express your answer in standard form.

$$\frac{2^8 \times 5^2}{2^2}$$

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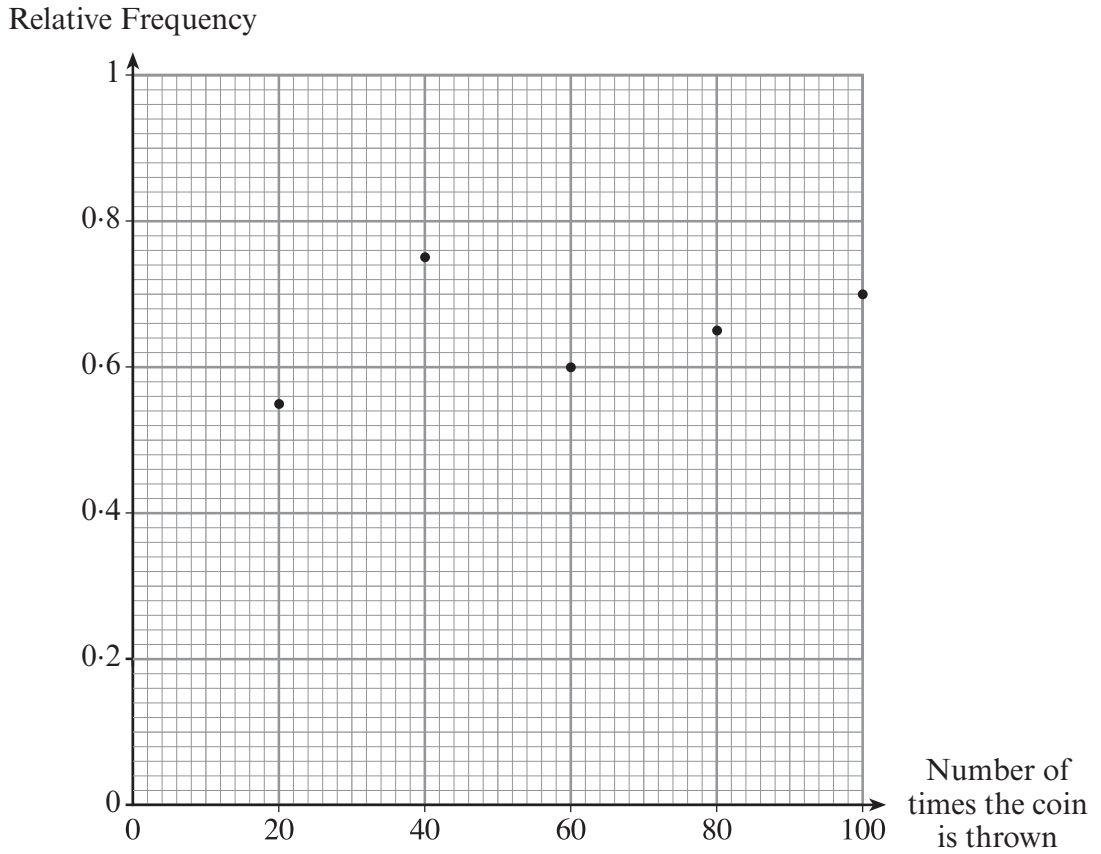
[3]

10. An experiment was carried out to investigate the probability of obtaining a tail when a biased coin is thrown.
The relative frequency of a tail was calculated after throwing the coin a total of 20 times, 40 times, 60 times, 80 times and 100 times.
The results are plotted on the graph below.

Relative Frequency

Number of times the coin is thrown

(a) Which reading do you think gives the best estimate for the probability of obtaining a tail? You must give a reason for your answer.

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[2]

(b) Using the graph, find how many

(i) tails were obtained in the first 60 throws,

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(ii) heads were obtained in the 100 throws.

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[4]

11.
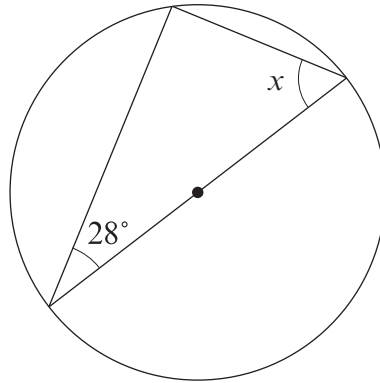
Diagram not drawn to scale

Find the size of the angle marked x.

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[2]

12. Given that y is inversely proportional to x^2, and that $y = 4$ when $x = 5$,

(a) find an expression for y in terms of x,

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[3]

(b) use the expression you found in *(a)* to complete the following table.

x	−1	5	
y		4	1

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[2]

Turn over.

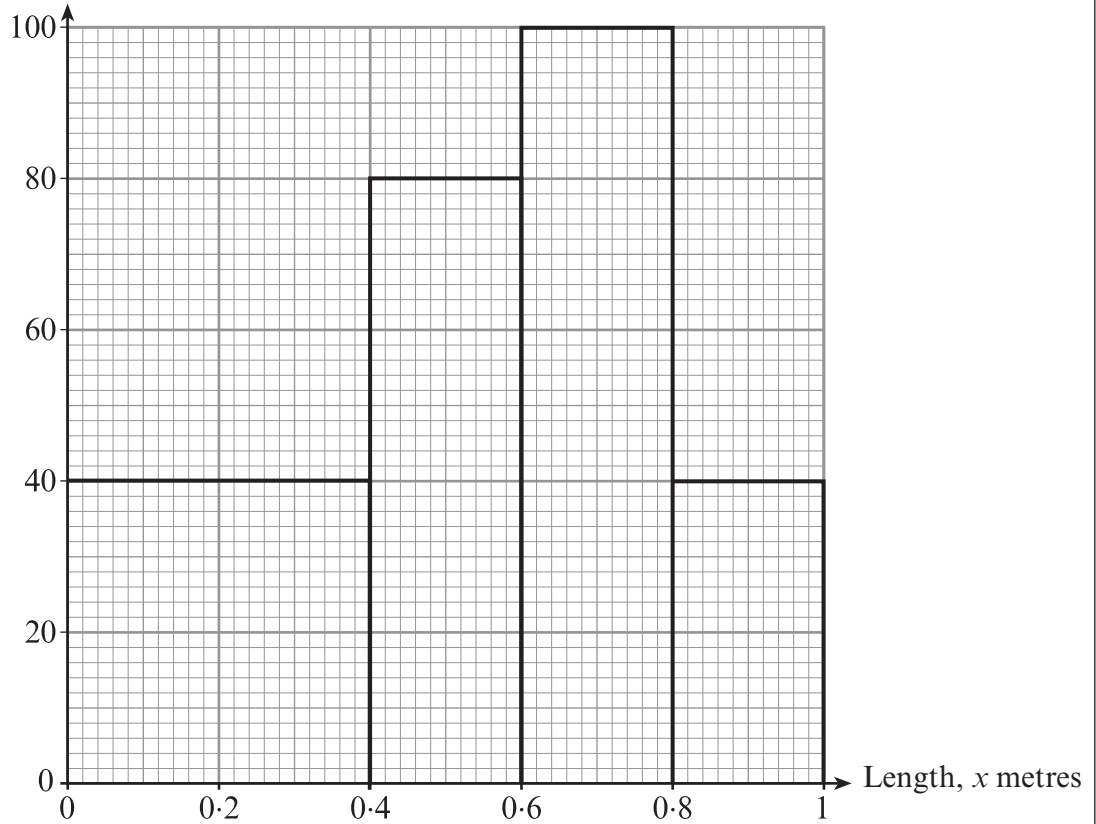**13.** *(a)* The lengths of logs in a pile were recorded.
The histogram below illustrates the results.

Frequency density

(i) Use the histogram to complete the grouped frequency table below.

Length, x metres	$0 < x \leqslant 0.4$	$0.4 < x \leqslant 0.6$	$0.6 < x \leqslant 0.8$	$0.8 < x \leqslant 1$
Frequency				8

(ii) Calculate an estimate of the number of logs with lengths greater than 0·3 m.

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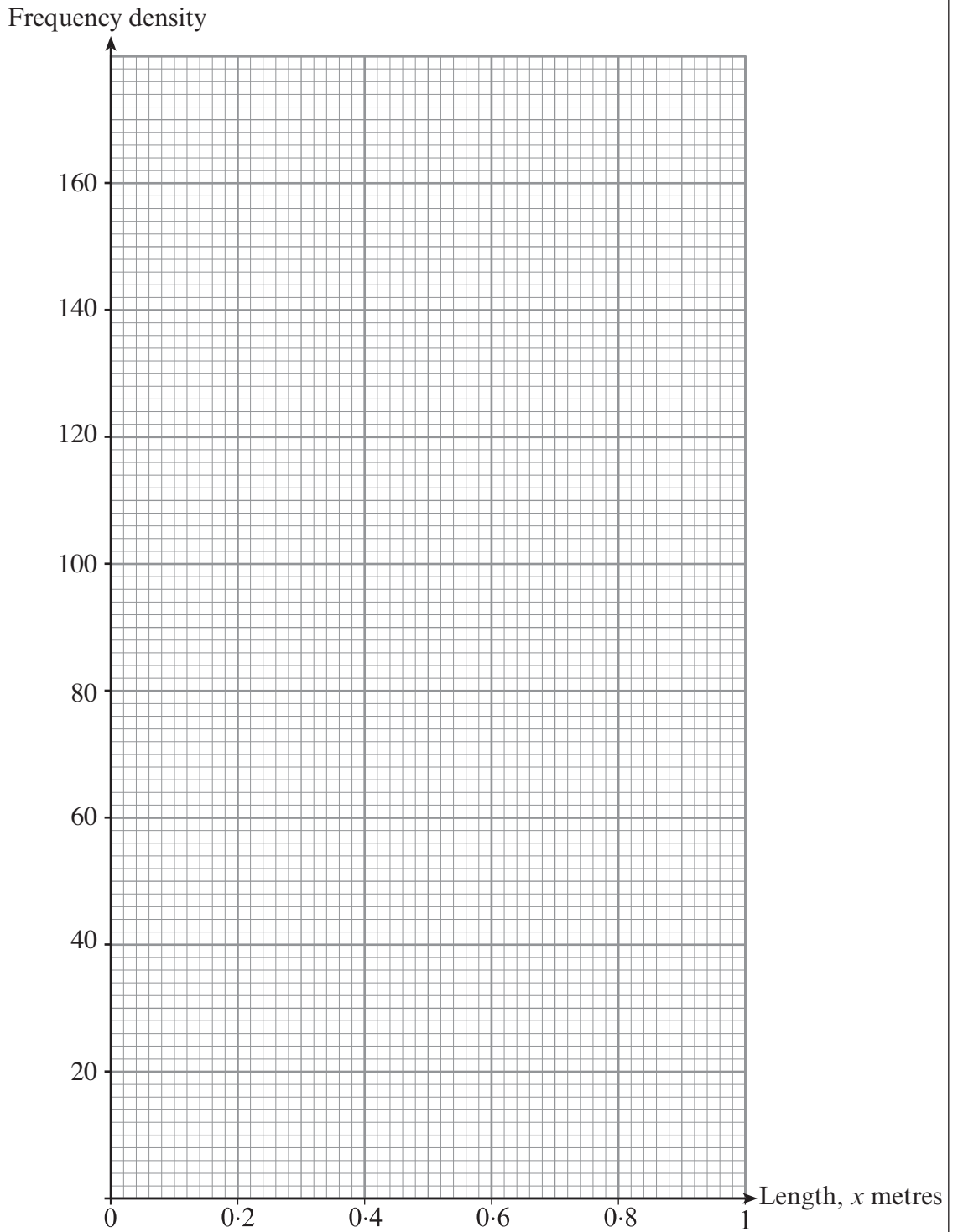
[4]

(b) The lengths of logs in a different pile were recorded.
The results are summarised in the grouped frequency distribution below.

Length, x metres	$0 < x \leqslant 0.4$	$0.4 < x \leqslant 0.6$	$0.6 < x \leqslant 0.8$	$0.8 < x \leqslant 1$
Frequency	16	30	8	6
Frequency density				

Complete the frequency density row in the table and draw a histogram.

[3]

Frequency density

Length, x metres

(c)   Thomas compares the two histograms.
He states that the mean length of the logs in the first pile is greater than the mean length of the logs in the second pile.
Is Thomas correct? You must give a reason for your answer.

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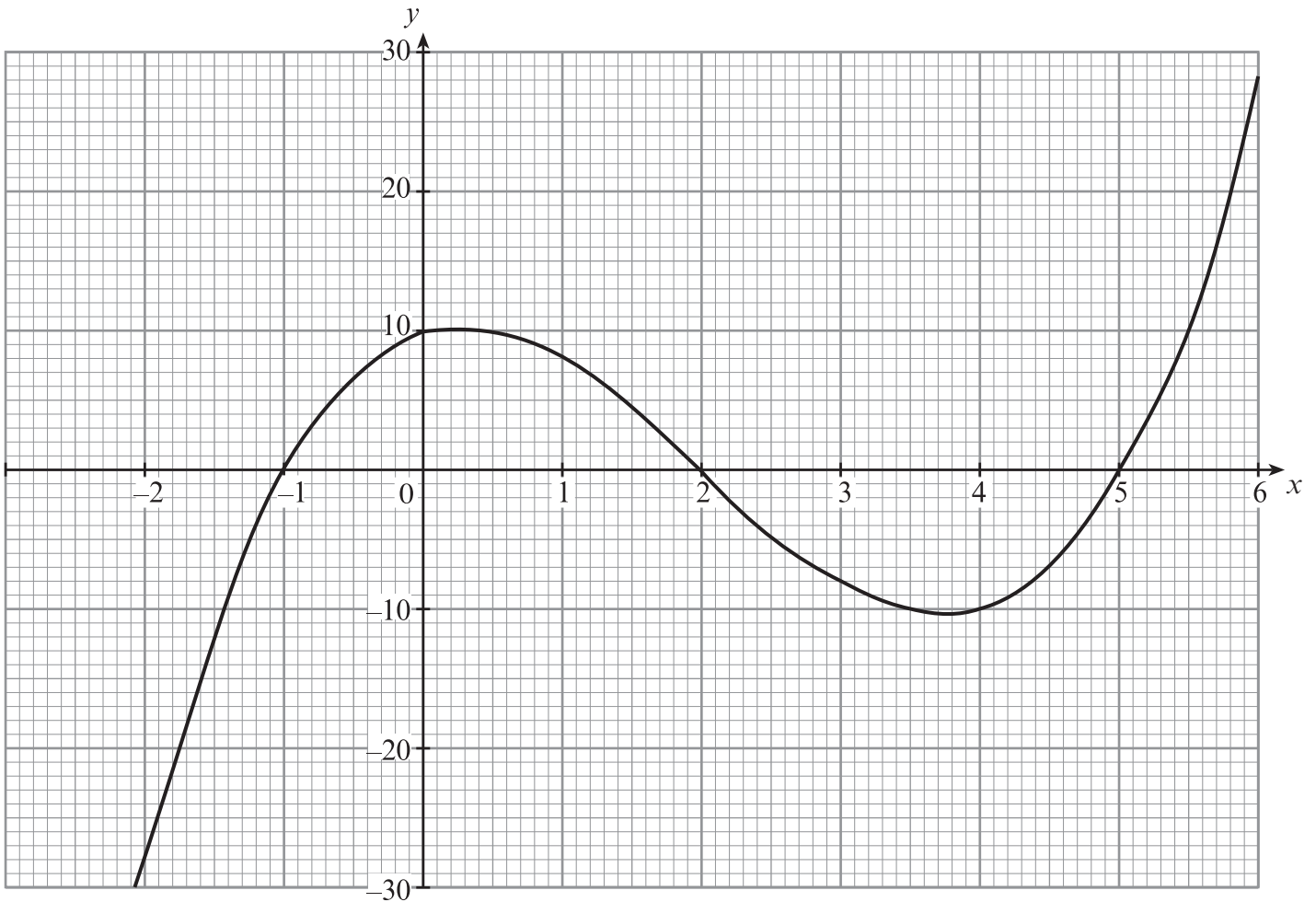
[2]

Turn over.

14. The graph of $y = x^3 - 6x^2 + 3x + 10$, for values of x between $x = -2$ and $x = 6$, is drawn below.

(a) Use the graph to solve $x^3 - 6x^2 + 3x + 10 = 0$.

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[1]

(b) Using the graph, estimate the gradient of the curve $y = x^3 - 6x^2 + 3x + 10$ when $x = 4\cdot5$.

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[3]

(c) By drawing an appropriate straight line on the graph, solve the equation

$$x^3 - 6x^2 + 2x + 10 = 0$$

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[3]

(d) Use the trapezium rule with three strips of equal width to estimate the area of the region enclosed by the curve $y = x^3 - 6x^2 + 3x + 10$ and the *x*-axis between $x = -1$ and $x = 2$.

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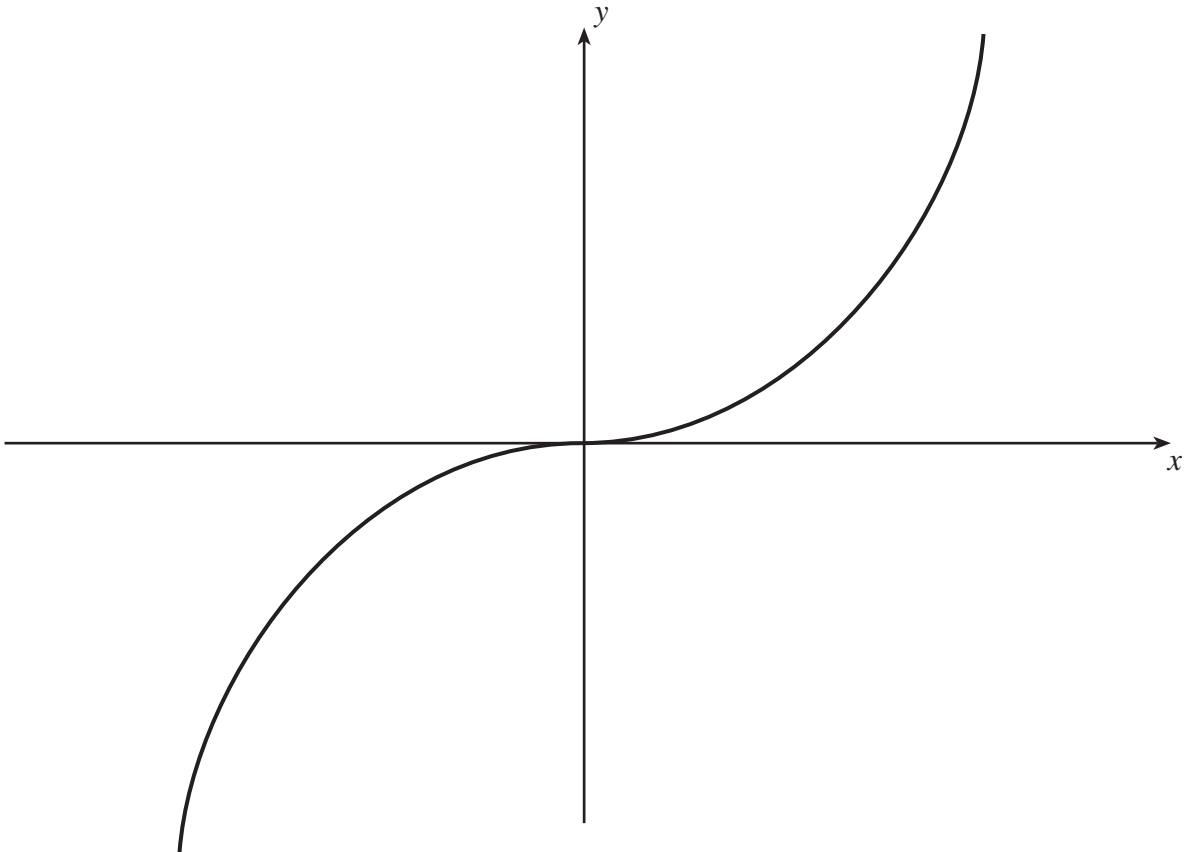
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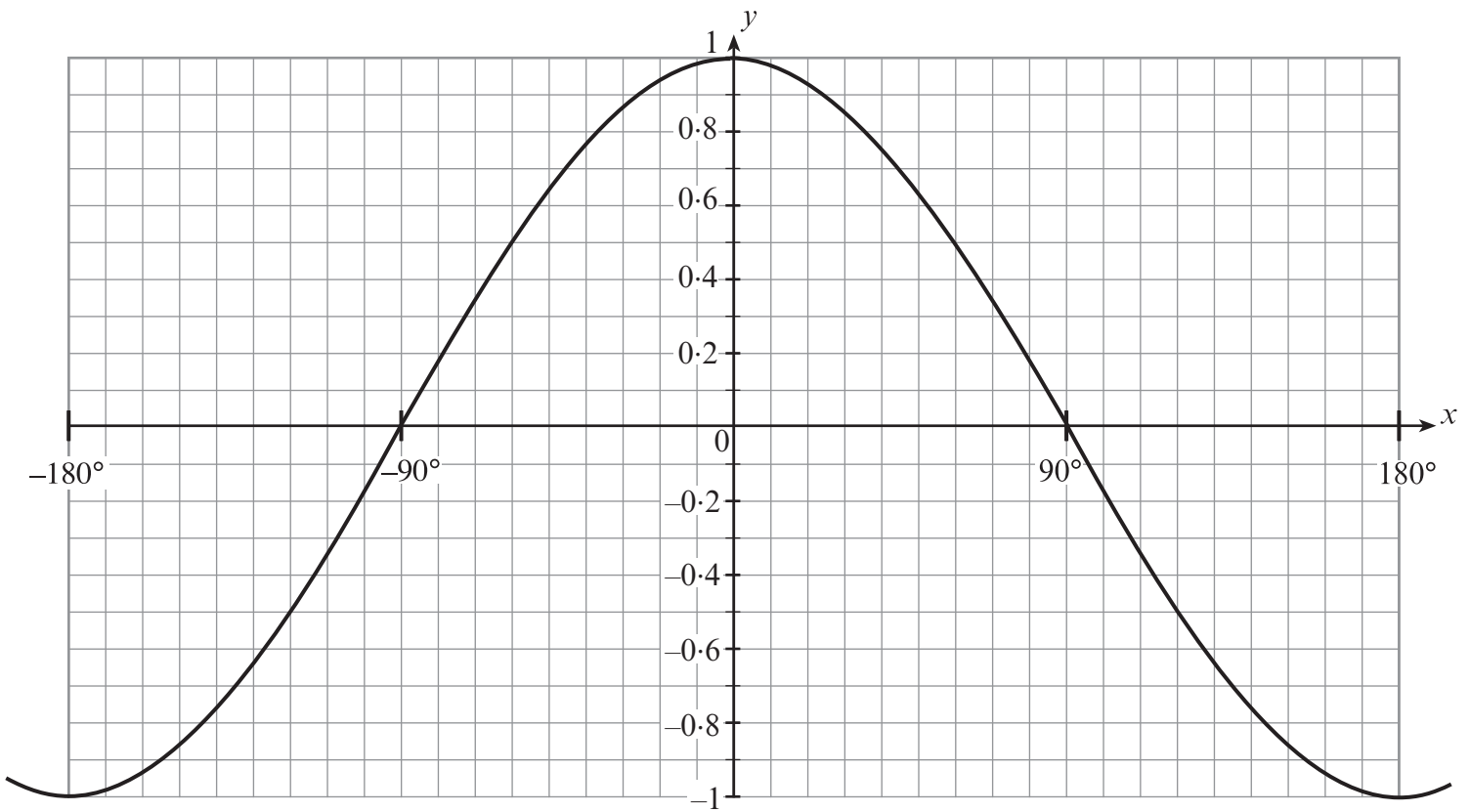
[4]

Turn over.

15. *(a)* The diagram shows a sketch of $y = x^3$.
On the same axes draw a sketch of $y = -4x^3$.

[2]

[2]

(b) The diagram shows a sketch of $y = \cos x$.

Find the values of x in the range $-180° \leqslant x \leqslant 180°$ which satisfy the equation $\cos x = 0.5$.

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[2]